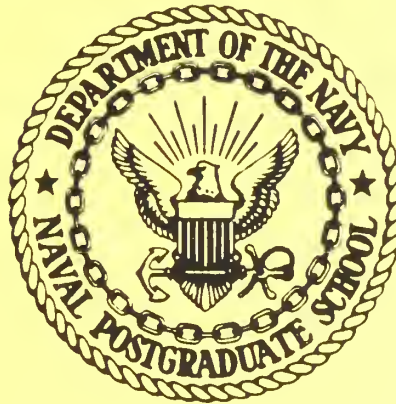


NPS55-88-006

NAVAL POSTGRADUATE SCHOOL

Monterey, California



THE USE OF SHRINKAGE TECHNIQUES IN
THE ESTIMATION OF ATTRITION RATES
FOR LARGE SCALE MANPOWER MODELS

BY

R. R. READ

JULY 1988

Approved for public release; distribution is unlimited

Naval Postgraduate School
Monterey, CA 93943

Naval Postgraduate School
Monterey, California

Rear Admiral R. C. Austin
Superintendent

H. Shull
Provost

This report was prepared in conjunction with research conducted for Headquarters, USMC and funded by the Naval Postgraduate School.

Reproduction of all or part of this report is authorized.

This report was prepared by:

1a REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b RESTRICTIVE MARKINGS MONTEREY CA 93943-5101		
2a SECURITY CLASSIFICATION AUTHORITY			3 DISTRIBUTION AVAILABILITY OF REPORT Approved for public release; distribution unlimited		
2b DECLASSIFICATION/DOWNGRADING SCHEDULE					
4 PERFORMING ORGANIZATION REPORT NUMBER(S) NPS-55-88-006			5 MONITORING ORGANIZATION REPORT NUMBER(S)		
6a NAME OF PERFORMING ORGANIZATION Naval Postgraduate School		6b OFFICE SYMBOL (If applicable) Code 55		7a NAME OF MONITORING ORGANIZATION Headquarters, USMC	
6c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000			7b ADDRESS (City, State, and ZIP Code) Washington, DC 20380		
8a NAME OF FUNDING/SPONSORING ORGANIZATION Naval Postgraduate School		8b OFFICE SYMBOL (If applicable) MPI-45		9 PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER Direct Funding	
8c ADDRESS (City, State, and ZIP Code) Monterey, CA 93943			10 SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO	PROJECT NO	TASK NO
					WORK UNIT ACCESSION NO
11 TITLE (Include Security Classification) THE USE OF SHRINKAGE TECHNIQUES IN THE ESTIMATION OF ATTRITION RATES FOR LARGE SCALE MANPOWER MODELS					
12. PERSONAL AUTHOR(S) R. R. Read					
13a. TYPE OF REPORT Technical		13b. TIME COVERED FROM _____ TO _____		14 DATE OF REPORT (Year, Month, Day) 88 July 27	
15 PAGE COUNT 38					
16 SUPPLEMENTARY NOTATION					
17 COSATI CODES			18 SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP			
19 ABSTRACT (Continue on reverse if necessary and identify by block number) This report summarizes the research to date performed by the author and his students on the use of modern multiparameter estimation techniques in the building of an attrition rate generator in support of the USMC Officer Planning and Utility System (OPUS). Three main areas are identified: The cell aggregation problem; the specifics of parameter estimation; the need to match forecasting techniques to the specific application. Most of the effort has been in the first two of these areas and much has been learned. The aggregation problem, i.e., the grouping of personnel cells into an appropriate number having common size and attrition behavior has emerged as the most important problem that requires immediate attention. Its resolution is expected to lead to a clear policy for multiparameter estimation. Estimation and forecasting are both impacted by the nature of the data base. It is likely that specific applications will use differing data bases and differing statistical techniques as well.					
20 DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT <input type="checkbox"/> DTIC USERS			21 ABSTRACT SECURITY CLASSIFICATION UNCLASSIFIED		
22a NAME OF RESPONSIBLE INDIVIDUAL R. R. Read			22b TELEPHONE (Include Area Code) 408-646-2382		22c OFFICE SYMBOL Code 55Re

THE USE OF SHRINKAGE TECHNIQUES IN THE ESTIMATION OF ATTRITION RATES FOR LARGE SCALE MANPOWER MODELS

R. R. Read

Department of Operations Research
Naval Postgraduate School
Monterey, CA 93943-5000

Abstract

This report summarizes the research to date performed by the author and his students on the use of modern multiparameter estimation techniques in the building of an attrition rate generator in support of the USMC Officer Planning and Utility System (OPUS). Three main areas are identified: The cell aggregation problem; the specifics of parameter estimation; the need to match forecasting techniques to the specific application. Most of the effort has been in the first two of these areas and much has been learned. The aggregation problem, i.e., the grouping of personnel cells into an appropriate number having common size and attrition behavior, has emerged as the most important problem that requires immediate attention. Its resolution is expected to lead to a clear policy for multiparameter estimation. Estimation and forecasting are both impacted by the nature of the data base. It is likely that specific applications will use differing data bases and differing statistical techniques as well.

EXECUTIVE SUMMARY

The author and his students have been working with a number of modern techniques applied to the problem of estimating attrition (leave the service) rates for the numerous cells that appear in manpower planning models for the USMC officer corps. Special attention is given to the "small cell" problem; that is, officer categories that normally contain but a few personnel. These cells are numerous and historical empirical rates for them are generally quite unstable. This report will summarize what we have learned to date, and outline the research continuation plans.

Work that has applied shrinkage type estimators to the problem of estimating officer attrition rates has been reported in [22,50,57]. The methods tested have been successful in the comparative sense. That is, they perform better than the raw historical rates that might be used in an ad hoc fashion. But their behavior in the absolute sense still has erratic aspects. Moreover we do not have a solid way to anticipate the areas of unstable performance.

The recent acquisition of a much more refined data tape and the theses by Larsen and Dickenson [38,22] have lead to greater insight to this problem. In particular the new data covers ten years, breaks out officer grade by above zone and in or below zone, regular and reserve, unrestricted and limited duty, etc. The thesis by Larsen identifies the important break points in the YCS (years of commissioned service) scale and some MOS (military occupation specialty) categories that must be treated separately. The thesis by Dickinson, in addition to pursuing some isolated details that had been treated presumptuously in earlier work, introduces an empirical Bayes method that appears to be doing a better job of shrinking the raw estimates. It seems to manage better the unevenness of the cell inventories. Finally, some of our problems have also been experienced by Carter and Rolph [13] so we propose to pursue their suggestions as well.

Our studies have led us to believe that the most important item in the continuation work is the aggregation problem. This problem has two aspects:

- (i.) The grouping of cells into communities of homogeneous attrition behavior.
- (ii.) The combining or amalgamation of cells in order to meet minimal cell inventory requirements.

This need involves some exploration of the data. Because of the cumbersomeness of the data extraction problem, it will be necessary to make wise choices and study the most germane collections of cells.

Based upon the results of Carter and Rolf, we anticipate that an adequate solution to the aggregation problem will lead to a clear policy for attrition rates generation. Once this is accomplished, we can turn to the specific needs and idiosyncrasies of the various application models. This will include questions of both short term and long term forecasting.

1 INTRODUCTION

In recent years, the Marine Corps has been phasing its manpower management into a centrally organized and computerized Officer Planning and Utility System (OPUS) [15, 16,17,18,19,20]. This system contains a number of planning models and such models are affected by three general factors: existing inventory (personnel), projected losses, and projected gains. In order to project the inventory into various future time periods, it is necessary to use a realistic system of flow rates. Some of the rates are under administrative control, such as promotions, job assignments, and of course everyone acquires longevity with the passage of time. The attrition flowrates, however, can be anticipated only in a statistical sense. By attrition we mean leaving the service for any reason (e.g. resignation, discharge, disability, release, retirement) and the circumstances that lead to these attritions are not under the control of the planner. (Note: Some attritions are voluntary and some involuntary. For general purposes we assume the planner is not cognizant of the involuntary losses.)

Our role in support of OPUS is to develop useful attrition rates so that losses of this type can be reasonably estimated. Obviously, the replacement lead time for planning is seldom small; most replacements ascend into the service as young lieutenants; augmentation from the reserves is also used. The cost of poor planning is great. Too many planned replacements lead to under utilized personnel; too few lead to jobs unfilled and the inability to function as required.

The purpose of this report is to gather and summarize what we have learned about attrition rate generation as it pertains to the USMC officer corps; to describe the work in progress; to outline ways to study forecasting methods that can serve the individual needs of the various models. Thus sponsors and others are given current appraisal. This report also serves as a working document for students and other researchers. The terminology and notation are standardized.

The report is organized as follows: Following this introduction we lay the base in terms of details of the problem description, notation, conventions, data structure and estimation methods. This section will also include a number of satellite issues including a discussion of the measures of effectiveness and the validation techniques. Section 3 contains summaries of the seven theses [1,22,34,38,50,57,58] that have been written in support of this project and discusses how they integrate towards the common goal. Section 4 is devoted to a brief discussion of futuristics. It appears important that the researchers familiarize themselves with the needs of specific user manpower models. Data structures and forecasting methods should be tailored for them.

2 PROBLEM DESCRIPTION, ISSUES, DETAILS.

A. General Structure and Notation

For the macro view it is convenient to think of the officer “cells” as the result of cross classifying according to grade (GR), military occupation specialty (MOS) and length of service (LOS). It will be seen later that further refinement

is useful and sharpens the results. (This will be discussed under Data and Conventions.) Some of these cells are large (i.e. have large personnel inventory), e.g. the grades of first lieutenant or captain with 3-7 years of service and in the combat arms occupations. Those familiar with the Corps realize that there are many, many small cells. The GR factor has a pyramid structure; fewer officers in the higher grades. Of course, GR is well correlated with LOS, but not sufficiently so that one of them can be removed from consideration. (Also LOS is closely correlated with YCS, years of commissioned service, and there are instances for which this distinction is important.) Under MOS we have considerable variability in that many officers are designated as qualified under several job codes. Some of the codes are robust in that there is a reasonable level of transferability; i.e. with a modest amount of training, an officer can transfer from one job to another. Other codes have high training costs or high levels of specialization; e.g. the aviation communities, and attorneys. Such considerations are very important to the manpower planner. They also impact upon the way that we build an attrition rate generator because the stabilization of rates for small cells will depend upon our ability to gather together small cells that have a communality of characteristics.

The time flow of personnel through the system involves gaining a year on the LOS scale each year, periodic advancements (or not) in GR, and changes in MOS (responsibilities increase with experience). The USMC normally has between 18,000 and 20,000 officers. Although there are dependencies in the cell flows we are not prepared to include them in the modeling process of the attrition aspects of such a large system. Instead, a binomial distribution model is adopted. Further we presume cell to cell independence. The impact of the independence assumption will be softened by the way that we aggregate cells, and by the estimation technique.

Although the cells are most numerous and their specifications are the result of cross classification, for purposes of study and development we assume that

homogeneous subsets of cells have been identified and, within each, they are placed in a lineal set. The letter k is used generally to represent the number of cells in a set; the letter T represents the number of years data to be used in the estimation process. Thus for $i = 1, \dots, k$ and $t = 1, \dots, T$, let

$$N_i(t) = \text{inventory of cell } i \text{ in year } t; \quad (1)$$

$$Y_i(t) = \text{number of attritions in cell } i \text{ in year } t. \quad (2)$$

Basically the raw empirical attrition rate for cell i is the maximum likelihood estimator (MLE)

$$\hat{p}_i = \left(\sum_t Y_i(t) \right) / \left(\sum_t N_i(t) \right) \quad (3)$$

This works well for large cells, but not for small ones. (E.g. The information in 0/5 is considerably different from that in 0/500, yet the MLE is the same.) The overall attrition rate for USMC officers averages about 10% in recent years. Thus our statistical “small cell” problem is compounded by a “low rate” problem.

The overall strategy for addressing our problem has two main aspects. They will be called the aggregation problem and the shrinkage method problem. There are a rather large variety of ways to manage each and it appears that they cannot be treated in isolation, but must be managed together.

The aggregation problem was stated earlier and we repeat it now. We have spoken of collections of homogeneous subsets of cells that possess a communality of behavior with respect to attrition. For our purposes we must emphasize two facets to this problem:

1. The identification of adequate numbers of cells whose inventory personnel are likely to have common attrition behavior.
2. The grouping together or amalgamation of the small cells in the aggregate into super cells whose inventory values meet minimal requirements, specified by the user.

Previous rate generators have been concerned only with 2, because 1 has no role if one uses historical rates. The advantages of shrinkage methods comes from the use of information contained in similar cells. The key is to identify the similar cells. Hence, item 1 is included in our aggregation problem.

The shrinkage method problem is involved with procedures which, for a given aggregate, must choose a single central rate for that aggregate and shrinkage factors which shrinks the cell MLE, \hat{p}_i , towards the central value by an amount equal to that cell's shrinkage factor.

In the work to date the shrinkage problem has received the greater attention [22,50,57]. In the last decade or two the statistical literature has displayed many papers on shrinkage methods for multiparameter estimation problems. The results, in the light of applications, have been startling and glamorous, see e.g.[13,27,29]. Naturally, it has been more exciting (and easier) to try these methods on our attrition rate problem using ad hoc, but defensible, cell aggregations.

On the other hand the aggregation problem has not been totally ignored. But it has proved to be more difficult, largely because of the cumbersome data handling problems. The theses by Elseramegy and Larsen [1,38] have dealt with this problem. An important observation by Carter and Rolf [13; 882-3] is that the inventory numbers for the cells in an aggregation should not be highly variable. This principle was not used in choosing the ad hoc aggregates mentioned in the preceding paragraph.

Returning to the question of shrinkage methods, there is an important general point that should be made at this time. Most of the methodological development has used the mathematical setting of independent normal random variables with common variance, see e.g.[22,23,24, 26,27,36]. Moreover, several applications [13,27] have been successful using binomial data which has been transformed to behave more like normal data. Thus our approach to shrinkage estimation has followed this lead and has three steps:

1. Transform the raw cell data via the Freeman Tukey transformation [27,30].
2. Apply the shrinkage method to the transformed data.
3. Invert the results to the original scale.

The Freeman-Tukey transform is an enhancement of the basic arc sin transform for binomial data which is designed to give more stability to the variance and make the distribution closer to normal. The form that we have been using is (dropping the cell and time affixes)

$$X = \frac{1}{2}\sqrt{N + .5} \left\{ \arcsin \left(2\frac{Y}{N+1} - 1 \right) + \arcsin \left(2\frac{Y+1}{N+1} - 1 \right) \right\}, \quad (4)$$

where N is the cell inventory and Y is its leaver count. This form appears to be different from the more customary

$$\sqrt{N + .5} \left\{ \arcsin \sqrt{\frac{Y}{N+1}} + \arcsin \sqrt{\frac{Y+1}{N+1}} \right\}. \quad (5)$$

Both have variance approximately equal to one. Because of the identity

$$\sin^{-1}(2p - 1) \equiv 2\sin^{-1}(\sqrt{p}) - \pi/2, \quad (6)$$

they are effectively the same, differing only by the term $(\sqrt{N + .5}) \pi/2$. The former was chosen for use because it circumvents the computation of a large number of square roots. The shrinkage process is applied to the data X of eq.(4) after averaging over time and developing a collection of these values for all the cells in an aggregate. This is described in detail in the subsection C. There are also questions of detail concerning how to invert the result. These too will be deferred. For now, it suffices to recognize that the transform (4) is an average value for

$$\sqrt{N + .5} \arcsin(2p - 1) \quad (7)$$

and if X^* is the shrunken value for X then the shrunken value for p will look

like

$$\begin{aligned}
 &= 0 && \text{if } X^* < (-\pi/2) \sqrt{N + .5} \\
 p^* &= \frac{1}{2} \left\{ 1 + \sin \left(X^* / \sqrt{N + .5} \right) \right\}, && \text{otherwise} \\
 &= 1 && \text{if } X^* > (\pi/2) \sqrt{N + .5}
 \end{aligned} \tag{8}$$

A number of notational conventions have been adopted during the course of the project. They are used both separately and in concert. We conclude this subsection with a listing of them

TS	transformed scale	MO	mean overage
OS	original scale	MU	mean underage
ML	maximum likelihood	MAD	mean absolute deviation
JS	James - Stein	SSB	sum of squares between groups
LT	limited translation	SSE	sum of squared errors
EB	empirical Bayes	GR	grade
MOS	military occupation speciality	OF	occupation field
LOS	length of service	YCS	years of commissioned service
LDO	limited duty officer	UNR	unrestricted
MOE	measure of effectiveness	FOM	figure of merit

B. Data. Conventions

The original data tape supplied by NPRDC contained data for seven years, 1977 thru 1983. It was possible to identify 10 grades (warrant officer 0-3, second lieutenant - colonel), 31 LOS levels (0 - 30 years with the final one being 30 or more), 40 MOS levels (actually OF, the first two digits of the four digit MOS), and 8 loss types. Details appear in [57]. This was the data base used in the theses [1,34,50,57,58].

A more extensive and refined data tape was received in the summer of 1987. It contained 10 years of data, 1977 thru 1986. For a complete description of the refinements see [38]. For our immediate purposes it suffices to point out that LOS is replaced by YCS (31 cells); GR is further broken out by UNR/LDO and

the failed select (to promote) are separated from the others; full MOS codes are available; commissioning source (15 levels); education (4 levels); regulars can be separated from reserves.

It is important to draw attention to the distinction between central and transition data. See [3; p24]. This impacts upon the way that the data are used. For the earlier tape mentioned above, the cell inventories refer to specific dates (or “snapshot” data). It is the number of occupants of the cell at the beginning of the fiscal year. On the other hand, the attrition counts for a cell contain the number of leavers at any time during the year. If an officer changes cells during the year and then leaves, the attrition is credited to the cell occupied at the time of leaving, not the cell that credits him for inventory. As an extreme case of this situation it is possible for a cell to contain zero inventory and yet record several leavers.

For this reason the following convention was adopted. First the cell inventory is replaced by the average of the beginning and end of year inventories. (Note: this is possible for all years save the last, which must use only the initial figure). Second, the central inventory is defined to be the larger of the average inventory and the number of leavers. In this way we are assured that $Y \leq N$ and these are the $N_i(t)$ values used in all formulas.

For the refined data tape, a different situation exists. The inventory figures are recorded in units of man-quarters. In this case, for our yearly analysis, we use the man-quarter figure divided by four in all formulas.

C. Concepts of Shrinkage Estimation: Heuristics

Perhaps the most familiar setting for describing this idea is that of one way analysis of variance (ANOVA). Consider independent random variables $\{X_{ij}\}$ and the distributional model

$$X_{ij} \sim N(\mu_i, \sigma^2) \quad i = 1, \dots, k; \quad j = 1, \dots, n. \quad (9)$$

The goal is to estimate μ_1, \dots, μ_k . Stein [53] has shown that the obvious estimators

$$\hat{\mu}_i = \bar{X}_i = \frac{1}{n} \sum_{j=1}^n X_{ij} \quad i = 1, \dots, k \quad (10)$$

are inadmissible using the average squared error loss function

$$L(\mu, \delta) = \frac{1}{k} \sum_{i=1}^k [\delta_i - \mu_i]^2 \quad (11)$$

where the $\delta_i = \delta_i(x)$ are the estimating statistics. That is, he constructed functions $\delta_i(x) \neq \bar{X}_i$ that have smaller values of L ; i.e.

$$\sum_{i=1}^k [\delta_i - \mu_i]^2 < \sum_{i=1}^k [\bar{X}_i - \mu_i]^2$$

and the dominating functions, $\{\delta_i\}$ are convex combinations of the $\{\bar{X}_i\}$ and the grand mean $\bar{X} = \frac{1}{k} \sum_{i=1}^k \bar{X}_i$. That is

$$\begin{aligned} \delta_i &= (1 - sh)\bar{X}_i + sh\bar{X} \\ &= \bar{X} + (1 - sh)(\bar{X}_i - \bar{X}). \end{aligned} \quad (12)$$

where sh is the (yet to be specified) shrinkage factor. Equation (12) provides the structure for all estimators that utilized fixed shrinkage toward the grand mean.

Heuristically, we would want the shrinkage factor to be larger (close to unity) when the $\{\mu_i\}$ are nearly all the same; i.e. the departures of the δ_i from the grand mean should be small. By way of contrast, if the μ_i are highly variable then the $\{\delta_i\}$ should not shrink very far from the group means, $\{\bar{X}_i\}$. The traditional analysis of variance technique provides a way of measuring the relative variability of the μ_i and, from this, a value for the shrinkage parameter can be produced.

The ANOVA table customarily produces the two sums of squares

$$\begin{aligned} SSB &= n \sum_{i=1}^k (\bar{X}_i - \bar{X})^2 \\ SSE &= \sum_{i=1}^k \sum_{j=1}^n (X_{ij} - \bar{X}_i)^2. \end{aligned} \quad (13)$$

The former, sum of squares between groups, is proportional to the sampling variance of the $\{\bar{X}_i\}$ and the latter, sum of squared errors, is proportional to the estimator for σ^2 . Thus shrinkage should vary inversely with the ratio SSB/SSE . The recommended scaling is

$$sh = \min \left(\frac{(k-3)}{k(n-1)+2} \frac{SSE}{SSB}, 1 \right), \quad (14)$$

[50,eq.3.22] and [24,eq.(7.7)]. This form is equivalent to the use of the positive part of $(1 - sh)$ which has been shown to improve upon the original James-Stein shrinkage, [26]. It will occur to some that a much simpler procedure is available by merely performing the ANOVA test for $H_o : \mu_1 = \mu_2 = \dots = \mu_k$. If we accept H_o , then use $\delta_i = \bar{X}_i$ for all i and otherwise use $\delta_i = \bar{X}$. This “testimator” procedure is also inadmissible, [51].

Multiparameter estimation methods that shrink the individual group estimators toward some common central value have appeared rather extensively under the names of Bayes or empirical Bayes procedures. Such procedures utilize some model enhancements for the data gathering process and these need to be reviewed in the light of each particular application. Since our applications involve binomial and multinomial probabilities, the reader is referred to [6; Chp.12] for methods and applications. For our application, a brief pilot study was made using these methods for the multinomial probabilities of the various attrition types. The results did not appear promising and we returned to our original course.

From a theoretical point of view we are engaged in an interesting conundrum. Having adopted the model of a large number of independent binomial cells, we know that there can be no Stein effect because the maximum likelihood estimator is admissible. This is true both for squared error loss [35] and the “chi square statistic” loss function, [48; p284]. Thus the justification of using shrinkage appears to come from the empirical Bayes arena. Yet our first attempt to use empirical Bayes directly was not at all encouraging.

The following is our interpretation of the riddle. The developed empirical Bayes procedures use beta function (or Dirichlet) prior distributions. These are the conjugate priors that facilitate the calculations. It is known that this sub-family does not perform well compared to maximum likelihood estimates when the cell probabilities are extreme (close to zero or one). Thus our lack of success is probably due to the fact that the attrition rates are small: overall longterm average of about 10% per year. Thus our encouraging results are credited to the idea that the basic strategy (transform, shrink, invert the transform) corresponds to an empirical Bayes procedure in an implicate way, see [21] for a general discussion. Others have had success doing this with a binomial setting, [13,27]. The fact that the binomial distribution is well approximated by a normal distribution, surely plays a role. Also, one should consider the thoughts of Berger,[5].

Lastly, we must keep in mind the weaknesses of our model. Perhaps the most important point here is the unlikelihood of year to year stationarity. The ultimate validation must somehow model and make reasonable allowances for these temporal changes. The mixing of “snapshot” and central data is also a problem, but we believe this is largely one of noise rather than one of structure. The independent cell binomial model, although thought to be robust, could be improved upon using general flow models. These latter models would be much more cumbersome to use on such a very large scale.

D. Loss Functions. Measures of Effectiveness. Validation

Several loss functions or measures of effectiveness (MOE's) have been applied in this project. Each serves its own purposes. A disquieting aspect of the research to date has been the fact that an estimation technique that works well with one MOE may make a poor showing using another one.

Initially we applied the James-Stein estimator. This estimator was designed

to perform well for normally distributed data using the squared error loss function

$$L(\delta, \mu) = \frac{1}{k} \sum_{i=1}^k (\delta_i - \mu_i)^2 \quad (15)$$

where $\delta = (\delta_1, \dots, \delta_k)$ are the estimating statistics and $\mu = (\mu_1, \dots, \mu_k)$ are the means to be estimated. (For validation purposes μ_i is replaced by the transformed data for the i^{th} cell during the validation year). Thus, this MOE was used to compare estimating schemes in the transformed scale, that is, after transforming and shrinking, but before inverting the shrunken estimates back to the original scale. These MOE values are identified by the words “transformed scale” (TS) squared error loss. They serve the purpose of measuring how well the “shrinkers” are performing compared to that specified by the supporting theory. The transform is scaled to produce a variance of unity, so we are looking for values of L near one.

Since the manpower planner cares little about performance on the transformed scale, and does care greatly about performance on the original scale, comparisons were also made using chi square statistics:

$$X_{(k)}^2 = \sum_{i=1}^k \frac{(a_i - e_i)^2}{n_i p_i (1 - p_i)} \quad (16)$$

where e_i = estimated number of attrition in the i^{th} cell; a_i = actual number of attritions in the i^{th} cell for the validation year; n_i = inventory for the i^{th} cell in the validation year; $p_i = e_i/n_i$. If the model is correct and the estimators are doing their job, this measure has a chi square distribution with k degrees of freedom. This fact means that its expected value is k , its variance is $2k$, and an absolute standard is available. There is a deceptive point, however. In a number of instances there are cells with non zero values for a_i and yet the maximum likelihood estimator, p_i , is either 0 or 1. In such cases the denominator of (16) is zero and the MOE cannot be computed. Rather than allow the information from the entire aggregate to be lost, we adopted the expedient of truncating the number of cells; k is reduced to k' (the number of useable cells) and the MOE

is computed and printed. This expedient has the effect of giving an unnatural advantage to the maximum likelihood estimators. The reader must interpret the results in the light of this point. No such truncation is applied for the competing shrinkage estimators, so comparisons become more difficult.

Discussion with NPRDC personnel over the MOE questions raised the issue that the chi square MOE is really a weighted squared error loss MOE that was chosen for its statistical properties. A measure is needed that is of more direct service to the manpower analyst. These thoughts have led to the recognition that (i) an average magnitude of errors is more useful, and (ii) the cost of overestimating is not the same as the cost of underestimating even if the magnitudes are the same. Since actual costs are not available and are likely to change among the aggregates, we adopted a general purpose method that allows the user to consider the magnitudes of underage and overage separately:

$$MO = \frac{1}{k} \sum_1^k (e_i - a_i)^+ \quad MU = \frac{1}{k} \sum_1^k (a_i - e_i)^+ \quad MAD = MO + MU \quad (17)$$

when MO stands for mean overage; MU for mean underage; MAD for mean absolute deviation; and the plus superscript denotes the positive part.

Unlike the previous two MOE's, we have no theoretical way to judge the adequacy of estimation schemes using (17). Thus one should prepare to compute some empirical savings figures. Letting $e_i(c)$ denote the attrition estimates for the i^{th} cell using current methods; $e_i(*)$ for proposed methods; and using these values to produce $MO(c), MO(*), MU(c), MU(*)$ one can then compute some relative savings figures

$$MO(*)/MO(c) \quad \text{and} \quad MU(*)/MU(c) \quad (18)$$

in order to make judgments about proposed procedures.

In summary then, we are looking for transformed scale loss figures of about one, original scale chi square figures of about k , and the best looking set of ratios for savings in underage and overage without having any absolute figure as a goal.

The details of validation require that designations be made for which data are used for developing the attrition estimates, e , and which are reserved for validation to provide the actuals, a . In the earlier works, it was arbitrarily decided to use the first four years (77-80, first data tape) for estimation and the last three (81-83) for validation [22,50,57]. The results indicate that the validation for 82 and 83 (two or more years into the future) are uniformly poor. It was concluded that there must be a time series effect and that questions of forecasting must ultimately be faced. More importantly, it was decided for immediate work to base all comparisons and conclusions upon the 1981 validation figures, (one year into the future).

A complete cross validation [28,55] is being planned for the empirical Bayes estimator, [Section 4.2]. The more refined (ten years) data tape will be used and each estimation calculation will use nine years of data. That is, each of the ten years will be taken out successively, case by case, for validation use while the remaining nine are used to develop the estimators. This is what we mean by a complete cross validation.

3 THESIS SUMMARIES

Seven Master's theses have been produced by this project. Each has made important contributions to the understanding of the problem. A brief summary of each will be given in this section, but the emphasis will be largely in terms of its bearing upon our two main problems: aggregation and estimation. On occasion, some of the important peripheral and supporting results will be mentioned, but lightly.

1. Tucker, D.D. [57]. This thesis is the initial one in the series. Major Tucker spent his experience tour at Headquarters USMC, used this opportunity for familiarization purposes, and did a superb job of obtaining background information and laying a proper base for others to work on the problem. His historical remarks, comments on the officer planning system, promotion prospects

by rank and coding of the structural zeroes (cells having zero inventory because of system structure) by MOS category are most useful. Further, he compiled a number of profile and other macro statistics that allow the researcher to envision how the system works. This thesis also contains the formatting information for the first data tape.

Major Tucker tested three estimation schemes; maximum likelihood (3), James Stein (12) and (14), and minimax. We display the minimax estimator here,

$$p_i^{(m)} = \frac{1}{1 + \sqrt{N_i(\cdot)}} \left[\frac{Y_i(\cdot)}{\sqrt{N_i(\cdot)}} + \frac{1}{2} \right] \quad (19)$$

where $Y_i(\cdot)$ and $N_i(\cdot)$ refer to $Y_i(t)$ and $N_i(t)$ summed over the estimation years. Explicit values of the average loss (11) appears in Table XVII,[57, p66].

This thesis used an ad hoc aggregation scheme which specified eight sets of officers; first lieutenants for each of four MOS groups and lieutenant colonels for each of the same four MOS groups. The MOS groups are:

1. Aviators (one OF code);
2. Ground Combat (three OF codes);
3. Combat Support (three OF codes); and
4. Combat Service Support (all others OF codes).

[57; p15]. Also all LOS cells were included which were not structural zeros when cross classified with GR and MOS.

The result of this study gave very substantial support to the James-Stein estimator. The minimax estimator was deemed to be too conservative for small cell use and was discarded.

2. Robinson, J.R. [50]. Based upon the work of Tucker, the immediate follow on effort was directed toward giving more attention to the small cells and a less hurried look at the basic James-Stein and maximum likelihood estimators. This was undertaken by Major J. R. Robinson, who also introduced the limited translations shrinkage alternative, see [24]; performed a more thorough

validation using both transformed scale, eq(15) and original scale eq.(16); and uncovered the fact that some of the arbitrary choices made earlier can have rather deep effects. Robinson also introduced the TSCA, transformed scale cell average, estimator.

This thesis used the same ad hoc aggregation scheme that was introduced by Tucker, except that the catchall aggregate, Combat Service Support, was dropped. The new TSCA estimator is computed by applying the Freeman-Tukey transformation (4) using as input the individual $N_i(t)$ and $Y_i(t)$, eq(1) and (2). The resulting $X_i(t)$ is then averaged over time. To invert to the original scale, one uses this value, call it X_i^* , together with $n_i^* =$ time average of inventory over the estimation years, and applies (8). Notice how this differs from the MLE, which averages over time prior to applying (4). Note further that TSCA may be viewed as James-Stein with zero shrinkage.

The limited translation James-Stein (LTJS) is complicated and the reader is referred elsewhere, [24] and [50; App.C], for details. We will however draw attention to some of its features. The basic idea is to reduce the amount of shrinkage in the tails of the distribution of the transformed values, X_i . This has the effect of reducing the individual errors for the extreme cells at the cost of (hopefully) only modest increases in total loss, eq.(11). To achieve this one is faced with the selection of a tuning constant, d , representing the number of standard deviations into the tails that one allows for full shrinkage before switching to reduced shrinkage. Robinson showed that this parameter, d , changed with the aggregated set. This author also studied some very small cells, i.e. inventory ranges (0,5) and (6,10).

The results of this thesis were sobering. First of all, the TSCA, MLE, JS, and LTJS estimators were all competitive. This was especially striking because in Tucker's work it appeared that JS was superior to MLE. Investigation into this matter showed that the method of counting cells in an aggregate can have a sharp effect. E.g. Tucker used the number of non structural zero cells whereas

Robinson used the number of non empty cells. Thus for example, the former used $k = 57$ and 48 for, respectively, ground combat first lieutenant and combat support lieutenant colonel; Robinson's values were $k = 45$ and 40 for these two groups. He also excluded the sampling zeros: cells of zero inventory because of sampling and not because of organizational structure. This change had the effect of returning MLE to competitiveness.

Earlier it was pointed out that MLE estimators allow values of zero and one, both of which make (16) uncomputable. When such values are removed in order to use the chi square measure, the values of k , for the above listed cells, becomes 35 and 23 . These facts dramatize our problem of cell definition and aggregation.

It was also discovered that Tucker's version of eq.(14), [57; p55, Step 3] is in error. In addition, Major Robinson's extensive study of the very small cells illustrated unstable behavior. That is, performance is at variance with that prescribed by theory. It may be better for the very small cells to be pooled together into single, larger cells rather than be exposed to this instability.

3. Amin Elseramegy, H.[1] This thesis reports our first attempt to treat the aggregation problem. The Naval Postgraduate School had recently acquired the very modern and glamorous CART (Classification and Regression Trees) program. Our plan was to try using this program to form aggregates of cells that exhibited homogeneity of behavior with regards to attrition, [1,9].

We ran into a number of difficulties, and the effort of learning to use the program became a major task. Our data base is much larger than that which the CART system provides for, as installed on our IBM 3033 system. It was necessary to partition it arbitrarily into nine sets so that each could be run separately. Moreover, to conserve computer memory, the LOS scale was treated as a quantitative interval scale and not as a set of categorical variables. Again the first four years were used for estimation, i.e., learning samples in CART parlance, and the raw attrition rate was used as the response variable.

Perhaps the point of greater import was that CART is a "top down" system.

It starts with all of the data (that memory can hold) in a single aggregate and goes through a succession of binary splits, each split making the most dramatic division possible on the scale of the response variable. A stopping rule terminates the process and the result is a binary tree. A new case can be dropped through the tree, follow the path prescribed by the succession of splits, and come to rest in a terminal node of the tree. That node will specify the attrition rate. This top down approach provided us with useful break points in the LOS (interval) scale in the earlier splits. The later splits were a mix and match set of GR and MOS combinations that had no apparent structure. Our applications require structure for customer oriented organizational purposes.

Thus the experience was useful in that it drew attention to the need for a “bottom up” approach to aggregation. Some organizationally meaningful cells should be brought together first. Then we must pool to get reasonably sized inventory numbers before computing response variables. We also learned that our ad hoc practice of using all (non structural-zero) LOS cells in an aggregate is a poor one.

4. Hogan, D.L.[34] Attention had been drawn to the fact that the validation figures for time lags of two and three years were poor and not used in the comparison of estimation schemes. That is, the values produced by the data (equally weighted) of four estimation years produced tenable values for the first year’s validation, but not for the other two years. This lead us to believe that there is a time series effect and Lieutenant Hogan explored the exponential smoothing technique, [11,34] in order to treat it.

In the large, this technique provides a way to update estimates yearly with the passage of time. It weights the recent past more heavily and discounts the distant past exponentially using a smoothing constant, α . Also, there is an interesting side advantage in that storage requirements are minimal.

Lieutenant Hogan worked with the four competitive estimators identified by Robinson, and the same six aggregates. The smoothing constant α was chosen

to minimize the MOE 's (or FOMs, figures of merit).

The results indicated that exponential smoothing does indeed give relief to the problem of estimating rates using larger time lags. The constant α , for the various aggregates, are larger than those generally encountered in other applications of exponential smoothing, and they are not as stable as we would like. In particular, the aviation community has emerged as being quite singular.

5. Yacin, N.[58] In response to intradepartmental pressures, it was decided to explore the logistic regression alternative using as carriers LOS (an interval scale) and GR (an ordered scale). Indeed, if successful the regression approach is preferred, [31,49,58].

Generally, but not always, shrinkage estimation methods (treating these variables as levels of two factors) perform better. The logistic regression made its best showing for $3 \leq LOS \leq 9$ and $4 \leq GR \leq 6$. Perhaps the most useful aspects of this study are the qualitative results:

- (i) For $0 \leq LOS \leq 3$: attrition rates are chaotic as young officers "test the waters".
- (ii) For $3 \leq LOS \leq 9$: attrition rates decline with increasing LOS as officers commit themselves to longer second and third contracts. One would think that advancement in grade would also correlate with a lower rate, but we don't see that. There appears to be other kinds of shifts influencing the attrition behavior in these years.
- (iii) For $9 \leq LOS \leq 19$: the maturing career commitment has been made and rates decline with increasing LOS and GR.
- (iv) For $19 \leq LOS \leq 30$: since advancement opportunities of the senior officer are quite limited we see rates increasing with LOS and decreasing with advances in GR.

6. Larsen, R.W.[38] Substantial progress in the aggregation problem was

made in this thesis. This is the first work that utilized the second, more refined, data tape. It contains the format for that tape. Captain Larsen presented a description of the current, dynamic (user specified threshold) aggregation method and followed the general plan specified by it. He applied a hierarchical clustering algorithm to the new data, see [2, 37, 38], and exposed the relative importance of some special MOS cells and YCS intervals. The separation of the aviation community into several groups is most revealing and undoubtedly explains much of the instability encountered earlier when the estimation schemes were applied to that group aggregated as a whole.

Equally important are the break points in the YCS scale uncovered by this thesis. Thus, a new order of putting cells together is indicated; a different set of priorities is established.

7. Dickinson, C. R.[22] This thesis also used the newer more refined data tape. We remind the reader that this tape recorded inventory in man quarters, whereas the previous one gave counts at the beginning of the fiscal year. This distinction appears to have a very noticeable effect. Also, LOS is replaced by YCS. Captain Dickinson repeated the Robinson calculations (MLE, TSCA, JS) for the same groups and included an empirical Bayes estimator as well. The results show that all are competitive in the comparative sense and the MOE numbers have greater stability than those exhibited using the other tape. Also, they are distinctly different from the earlier values.

In addition, Captain Dickinson performed some side studies treating issues that had been treated “out of hand” in earlier works. Specifically:

- (i) Approximate and use the unequal variances on the transformed scale.
- (ii) Study of the effect of alternative inversion formulae.
- (iii) Choice of inventory values for inversion of the transform.
- (iv) Graphical description of non uniform shrinkage and nonlinear shrinkage curves on the original scale.

To elaborate, item (i) is necessary in order to develop useful empirical Bayes estimators, [13,15,25,26,43]. Otherwise, the shrinkage is uniform for all groups and that situation should be adequately covered using the basic James-Stein estimator. Item (ii) deals with the question of returning to the original scale from the transformed scale. The basic inversion, eq.(8) has been used in all earlier studies, but some competitors have appeared in the open literature, [13,27,42]. Certainly, the FTE (Freeman-Tukey exact, Ref[42]) must be considered seriously since the basic inversion eq.(8), is necessarily only approximate. The problems encountered in this area are connected with those addressed in item (iii). The choice of inventory, n , to be used in the inversion varies with the group index. This leads to the awkward condition that full shrinkage to the grand mean on the transformed scale does not invert to a common attrition rate on the original scale.

Turning to item (iii), the FTE was discovered by Miller, [42], who also recommended the use of the harmonic mean (over time) when choosing an inventory value for purposes of inversion. Captain Dickinson studied this question via computer simulations using arithmetic, geometric, and harmonic means and the small values of attrition rates that are of interest to us. The arithmetic mean made the best showing, probably because of the small rates.

The graphical shrinkage paths, item (iv), are interesting, but not alarming. The individual paths are smooth and appear to have monotone derivations; the bow is not severe; straight line approximations would not be damaging.

In the eleventh hour of his work, Captain Dickinson experimented with a weighted empirical Bayes estimator, [22, App.E]. The result is very positive and this estimator is recommended for further study.

4 CURRENT STATUS AND CONTINUATION PLANS

1. Technical Insights

It has been known for a long time that the method of least squares (for our problem this means the use of raw empirical rates) picks up too much of the idiosyncracies of the “training” data set and leads to disappointing performance when used for predictions. There are a number of ways for combating this, many of them having an ad hoc flavor, and generally rather intensive in terms of computation. The class of James-Stein and other shrinkage methods possess very salable analytic support; their use should become wide spread.

The Elfron-Morris paper “*Data Analysis Using Stein’s Estimator and Its Generalizations*”, [27] presents reasons why more applications have not been forth coming. They also present three applications of the method that provide very dramatic improvements over classical methods, and serve as models for use by practitioners. Their toxomosis prevalence rate example is especially convincing. The data are completely real and the gains are of the order of 200 percent.

Their baseball example is a closer prototype to our application and the gains are given as 350 percent. This certainly appears attractive. The fact that the authors were able to practice some selectivity in this example has emerged as a point of importance along with the natural distinctions between batting averages, attrition rates and other aspects of our problem. We take a moment to discuss the insights that have been developed regarding these things.

In the batting average example 18 players were selected and the results of their first 45 times at bat were used for the estimation or training data set. The shrunken estimated batting averages were then compared with the end of season values, and with great success. The player selection scheme, [27; pg 312], was driven largely by the goal of exactly 45 times at bat on certain dates; all

rates exceeded .15. Thus, all $n_i = n$ and this value is sufficiently large so that the variance of the transform (essentially Freeman-Tukey), for $.15 \leq p \leq .85$, is constant. Moreover, this degree of selectivity also insures that there is no issue as to how to invert the transform after shrinkage.

For the attrition rate problem the vast majority of rates are below .15, the inventory values are seldom as high as 45 and certainly not constant. We have no guide as to how large k (number of cells) should be other than $k \geq 4$. Our experiences had led us to believe that unevenness in cell to cell (also over time) inventory is detracting from the performance of our estimators. Some isolated calculations have shown that the method of inversion, eq.(8), is an important issue. Thus our application breaks new ground and, when completed, will make an important contribution to the lore.

It appears that Carter and Rolph found similar issues. They state, [13, p382] paraphrased, that the empirical Bayes estimators will perform best if applied separately to groups (aggregates) of cells that have comparable size and similar rates. It is extremely interesting to note that their empirical Bayes estimators made their best performance (showed the greatest savings) for cells with low rates. This is also the experience of Fay and Herriott, [29].

Some of the other details of this Carter-Rolph paper are not clear. The transform inversion formula [13; pg 882], seems to have a misreferenced origin. As pointed out in [22] it appears to perform shrinkage towards $p = 0.5$ on the original scale. Since this detail interacts with the particular empirical Bayes method used, further guidance from this paper is not attractive.

Thus we believe that the ad hoc aggregates chosen for our pilot studies are detracting from our ability to discriminate among competing estimators. The next major effort should be a hands on study of the data following the lead of Larsen [38] and developing sensible aggregates that fit well with the natural organization of the USMC officer corps.

2. Current Estimation Recommendations

Empirical Bayes estimators require knowledge of the variance for each cell. On the original scale this is given by the familiar formula for the binomial distribution

$$Var(Y) = p(1 - p)/n \quad (20)$$

which can change sharply with both p and n . The situation is more pleasant on the transformed scale. Indeed the Freeman-Tukey transformation was designed to stabilize the variance at one, and it does so for the non extreme values of p . However in our problem there are important combinations of n and p for which the variance is smaller than one. Moreover we are fortunate in that a single interpolatory curve has been found that fits this variance function very well for broad combinations of n and p . Details appear in [22; App. C]; skeleton summary follows.

Let $\mu = E(X)$ when X is given by eq.(4). Then, to a very good approximation for $N \geq 3$,

$$Var(X) \doteq \max(1, V(\mu)) \quad (21)$$

where

$$V(\mu) = a(\mu - \pi/2)^{b_1}(\mu - 1 - \pi/2)^{b_2} \quad (22)$$

with

$$a = 1.6835 \quad b_1 = -.8934 \quad b_2 = .9881$$

and $\mu \geq 1.001 + \pi/2$. (Clearly the formula breaks down for $\mu - 1 - \pi/2$ negative.) The value one that appears in (21) dominates for (about) $\mu - \pi/2 > 2.2$. The formula (22) comes into play for $N > 2$, and $p \geq .001$, with the upper limit given by a function of N, p ; see [27].

Our policy for empirical Bayes estimation is described next. Consider a single cell and let T be the number of years in the estimation set. Then, using

$X(t)$ from (4), with the argument t inserted to denote the year, let

$$XT(t) = X(t)/\sqrt{.5 + N(t)} \quad \text{for } t = 1, \dots, T \quad (23)$$

except if $N(t) = 0$ then $XT(t)$ does not exist and T is reduced accordingly. Then form the time averages

$$XTB = \left[\sum_t XT(t) \right] / T \quad (24)$$

We require a single variance figure for the $XT(t)$, denote it VT , and define it implicitly by

$$Var(XTB) = \left[\sum_t Var(XT(t)) \right] / T^2 = VT/T \quad (25)$$

and (21) is used in the summand of (25) with μ replaced by $XT(t)$. Thus XTB is a time average of transformed values for the cell and VT is our estimate of its population variance.

Now the empirical Bayes value for our cell is the convex combination

$$XEB = \frac{A}{A + VT} XTB + \frac{VT}{A + VT} XBB \quad (26)$$

where XEB , XTB and VT change from cell to cell within the aggregate; XBB is a single central value (weighted average) and A is the variance of the prior distribution of cell means. Both A and XBB must be estimated jointly using an iterative algorithm. The details are next.

Let k be the number of cells, as before, and we will attach subscripts ($i = 1, \dots, k$) to previously defined quantities that depend upon the cell. We will use A_0 for the “previous” value of A in our iterative algorithm and initialize with $A = 0$. First set

$$A_0 \leftarrow A \quad (27)$$

Next define temporary values $\{\alpha_i\}$ and $\{\gamma_i\}$ by means of

$$\alpha_i = 1/(A + VT_i)$$

$$\gamma_i = \alpha_i / \sum_1^k \alpha_j$$

Then compute the weighted mean

$$A \leftarrow A - \frac{k - 1 - \sum_i \alpha_i [XTB_i - XBB]^2}{\sum_i \alpha_i^2 [XTB_i - XBB]^2} \quad (28)$$

Now, if the result is $A \leq 0$, set $A = 0$ and exit from the loop. Also if $|A - A_0| < \epsilon$ (say 10^{-3}), we are finished and should exit. Otherwise, return to (27) and repeat the steps.

Having determined XBB and A , we use these values in (26) to produce $XEB_i, i = 1, \dots, k$. Notice that the amount of shrinkage changes with the cell (i.e. VT_i are not necessarily all equal to one and if $A = 0$ then the shrinkage is 100 percent to XBB).

We pause to note that the previously tested non uniform shrinkage method (LTJS, [50]) selected cells with extreme time average values for reduced shrinkage. The empirical Bayes method chooses cells with lower variance for diminished shrinkage.

3. Forecasting

Often, the applications involve forecasting. There are great differences among the users as to the length of the forecast period. One application involves monthly forecasts while, at the other extreme, another involves yearly forecasts up to seven years into the future. The forecasting method should be tailored to the needs of the application. These are a number of techniques available, [7,8,10,34,41,52,56,].

Bres and Rowe [10] report success with Naval Officer attrition rate forecasting using a third order auto regressive model combined with a linear program that solves for the coefficients using MAD. But this success has diminished with time (Rowe, personal communication) and other techniques are being developed, [46,52]. Also, NPRDC is working with some econometric models.

The situation is different for very near term forecasting. Seventy percent of the yearly leavers do so in the summertime (Morton, DSI, personal communication). Often the users are experimenting with contingencies and sundry incentive plans. For such applications, Bayesian methods could prove useful, [7,17,34].

We have paid little attention to forecasting thus far in our project. The two and three year validations were abandoned because of their instability. The exponential smoothing applied by Hogan showed improvements but behaved erratically. We believe that a quality policy for managing the aggregation problem will do much toward laying the base to study forecasting. It appears that the blend of shrinkage estimation and multiparameter forecasting has yet to be treated in the open literature. This presents a challenge.

5 REFERENCES

1. Amin Elseramegy, H., *CART Program: Implementation of the CART Program and Its Application to Estimating Attrition Rates*, Masters Thesis, Naval Postgraduate School, Monterey, California. December 1985.
2. Anderberg, M.R., *Cluster Analysis for Applications*, Academic Press. 1973.
3. Bartholomew, David J. and Forbes, Andrew F., *Statistical Techniques for Manpower Planning*. Norwich, Great Britian: Wiley, 1979.
4. Berenson, M. L., Levine, and Goldstein, *Intermediate Statistical Methods and Applications*, Prentice Hall. 1983.
5. Berger, J. O., *Bayesian Robustness and the Stein Effect*, **Journal of the American Statistical Association**, v.77, pp.358-368. June 1982.
6. Bishop, Y., Fienberg, S. and Holland, P., *Discrete Multivariate Analysis: Theory and Practice*, Cambridge: MIT Press, 1975.
7. Bolstad, N. M., "*Harrison-Stevens Forecasting and the Multiprocess Dynamic Linear Model*", **The American Statistician**, v.40, pp.129-135. 1986.
8. Box, G. E. P. and Jenkins, G. M., "*Time Series Analysis: Forecasting and Control*", Holden-Day, San Fransico. 1976.
9. Breiman, Leo; Friedman, J. H.; Olshen, R. A.; Stone, C. J.; *Classification and Regression Trees*. Graybill, F., Wadsworth, Inc. 1984.
10. Bres and Rowe, *Development and Analysis of Loss Rate Forecasting Technique for the Navy's Unrestricted Line Officers*, Navy Personnel Research and Development Center. San Diego, California. 1985.
11. Brown, Robert Goodell, *Smoothing, Forecasting and Prediction of Discrete Time Series*. Englewood Cliffs, N.J.: Prentice-Hall. 1963.

12. Butterworth, R. W. and Milch, P. R., *Clustering Navy Ratings by Loss Behavior*, Naval Postgraduate School, Monterey, California. 1975.
13. Carter, G. and Rolph, J., *Empirical Bayes Methods Applied to Estimating Fire Alarm Probabilities*, **Journal of the American Statistical Association**, v.69, pp.880-885. December 1974.
14. Casella, G., *An Introduction to Empirical Bayes Data Analysis*, **Journal of American Statistical Association**, v.39, pp.83 -87. May 1985.
15. Decision System Associates, Inc., *User's Manual for the Officer Rate Generator*. Rockville, Maryland. 1985.
16. Decision System Associates, Inc., *OPUS-System Specification*. Rockville, Maryland. 1986.
17. Decision System Associates, Inc., *OPUS-System Specification for Optimun Officer Force Model*. Rockville, Maryland. 1987.
18. Decision System Associates, Inc., *OPUS- System Specifications for Officer Population Simulation*. Rockville, Maryland. 1986.
19. Decision System Associates, Inc., *User's Manual for Officer Planning and Utility System (OPUS)*. Rockville, Maryland. 1985.
20. Decision System Associates, Inc., *Functional Description for the Development of the Officer Planning and Utilization System (OPUS)*. Rockville, Maryland. 1986.
21. Deely, J. and Lindley, D., *Bayes Empirical Bayes*, **Journal of the American Statistical Association**, v.76, pp.833 - 841. December 1981.
22. Dickinson, C. R., *Refinement and Extension of Shrinkage Techniques in Loss Rate Estimation of Marine Corps Manpower Models*, Masters Thesis, Naval Postgraduate School, Monterey, California. March 1988.

23. Efron, B. and Morris, C., *Empirical Bayes on Vector Observations: An Extension of Stein's Method*, *Biometrika*, v.59, pp.335-346. 1972.
24. Efron, B. and Morris, C., *Limiting the Risk of Bayes and Empirical Bayes Estimators - Part I: The Bayes Empirical Case*, *Journal of American Statistical Association*, v.66, pp.807-815. December 1971.
25. Efron, B. and Morris, C., *Limiting the Risk of Bayes and Empirical Bayes Estimators - Part II: The Empirical Bayes Case*, *Journal of the American Statistical Association*, v.67, pp. 130-139. March 1972.
26. Efron, B. and Morris, C., *Stein's Estimation Rule and Its Competitors—An Empirical Bayes Approach*, *Journal of the American Statistical Association*, v.68, pp. 117-130. March 1973.
27. Efron, B. and Morris, C., *Data Analysis Using Stein's Estimator and Its Generalizations*, *Journal of the American Statistical Association*, v.70, pp. 311-319. June 1975.
28. Efron, B. and Gong, G., *A Leisurely Look at the Bootstrap, the Jackknife, and Cross-Validation*, *Journal of the American Statistical Association*, v.37, pp. 36-48. 1983.
29. Fay, R. E.III and Herriot, R. A., *Estimates of Income for Small Places: An Application of James-Stein's Procedures to Census Data*. *Journal of the American Statistical Association*, v.74, pp. 269-277. June 1979.
30. Freeman, M. F. and Tukey, J. W., *Transformations Related to the Angular and the Square Root*, *Annals of Mathematical Statistics*, v.21. 1950.
31. Gaver, D. and O'Muircheartaigh, I., *Robust Empirical Bayes Analysis of Events Rates*, unpublished paper, Naval Postgraduate School, Monterey, California. 1985.

32. Grinold, R. C. and Marshall, K. T., *Manpower Planning Models*, North Holland. 1977.
33. Harrison, P. S. and Stevens, C. F. *A Bayesian Approach to Short Term Forecasting*, *Operations Research Quarterly*, v.22, pp. 341-352.
34. Hogan, D. L. Jr., *The Use of Exponential Smoothing To Produce Yearly Updates of Loss Rates Estimates in Marine Corps Manpower Models*, Masters Thesis, Naval Postgraduate School, Monterey, California. June 1986.
35. Ighodaro, A. and Santner, T., *Ridge Estimators of Multinomial Cell Probabilities*, *Statistical Decision Theory and Related Topics III*, v.2, pp.31-53, ed. by Gupta, S. S. and Berger, J. O., Academic Press, New York. 1982.
36. James, W. and Stein, C., *Estimation with Quadratic Loss*, *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability*, v.1, Berkeley: University of California Press. 1961.
37. Johnson, S. C., *Hierarchical Clustering Schemes*, *Psychometrika*, v.32, pp.241-254. September 1967.
38. Larsen, R. W., *The Aggregation of Population Groups to Improve the Predictability of Marine Corps Officer Attention Estimation*, Masters Thesis, Naval Postgraduate School, Monterey, California. December 1987.
39. Lehman, E. L. and Hodges, J. L. Jr., *Some Problems in Minimax Point Estimation*, *Annals of Mathematical Statistics*, v.21, pp.182-197. June 1950.
40. McCullagh, P. and Nelder, J., *Generalized Linear Models*, Chapman and Hall. 1983.
41. McKenzie, E. *General Exponential Smoothing and the Equivalent ARMA Process*, *Journal of Forecasting*, v.3, pp. 333-344. 1984.

42. Miller, J. J., *The Inversion of the Freeman-Tukey Double Aresine Transformation*, *American Statistician*, v.32, p.138. November 1978.
43. Morris, C., *Parametric Empirical Bayes Inference: Theory and Application*, *Journal of the American Statistical Association*, v.78, pp.47-65. March 1983.
44. Mosteller, F. and Tukey, J. W., *Data Analysis and Regression: A Second Course in Statistics*, Addison-Wesley. 1972.
45. Navy Personnel Research and Development Center, *System Design for the Marine Corps Officer Rate Projector (MCORP)*, San Diego, California. 1985.
46. Navy Personnel Reserach and Development Center, *STRAP-O Five Year Verification/Validation Conclusions*. San Diego, California. November 1986.
47. O'Donovan, T. M., *Short Term Forecasting*. J. Wiley and Sons. New York. 1983.
48. Olkin, I. and Sobel, M., *Admissible and Minimax Estimation for the Multinomial Distribution and for k Independent Binomial Populations*, *Annals of Statistics*, v.7, pp.284-290. 1979.
49. Pregibon, D., *Logistic Regression Diagnostics Annals of Statistics*, v.9. November 1984.
50. Robinson, John R., *Limited Translation Shrinkage Estimation of Loss Rates in Marine Corps Manpower Models*, Masters Thesis. Naval Postgraduate School, Monterey, California. March 1986.
51. Sclove, S. L., Morris, C. and Radhakrishuan, R., *Non-optimality of Preliminary-Test Estimators for the Mean of a Multivariate Normal Distribution*, *Annals of Mathematical Statistics*, v.43, pp.1481-90. 1972.

52. Siegel, B., *Methods for Forecasting Officer Loss Rates*, Navy Personnel Research and Development Center, San Diego, California. 1983.
53. Stein, C., *Inadmissibility of the Usual Estimator for the Means of a Multivariate Normal Distribution*, **Proceedings of the Third Berkeley Symposium of Mathematical Statistics and Probability**, v.1. Berkeley: University of California Press, pp. 197-206. 1955.
54. Stein, C., *Estimation of the Mean of a Multivariate Normal Distribution*, **Annals of Statistics**, v.9, pp.1135-1151. November 1981.
55. Stone, M., *Cross-Validatory Choice and Assessment of Statistical Problems*, **Journal of the Royal Statistical Society, Section B**, v.36, pp.111-142. 1974.
56. Tiao, G. C. and Box, G. E. P., *Modeling Multiple Time Series with Applications*, **Journal of the American Statistical Association**, v.76, pp.802-816. 1981.
57. Tucker, D. D., *Loss Rate Estimation in Marine Corps Officer Manpower Models*, Masters Thesis. Naval Postgraduate School, Monterey, California. September 1985.
58. Yacin, Naci, *Application of Logistic Regression to the Estimation of Manpower Attrition Rates*, Masters Thesis. Naval Postgraduate School, Monterey, California. March 1987.

6 DISTRIBUTION LIST

	No. of Copies
Professor Peter Purdue, (Code 55Pd) Naval Postgraduate School Monterey, CA 93943-5000	1
Professor Robert R. Read, (Code 55Re) Naval Postgraduate School Monterey, CA 93943-5000	10
Marine Corps Representative, (Code 0309) Naval Postgraduate School Monterey, CA 93943-5000	1
Commandant of the Marine Corps HQMC, Code MPP-30 ATTN: Major Larsen ATTN: Major Maxfield Washington, D. C. 22134	2
Commandant of the Marine Corps HQMC, Code MPI-10 Washington, D. C. 22134	1
Commandant of the Marine Corps HQMC, Code MA ATTN: Major Tucker, Rm. 4023 Washington, D. C. 22134	1
Commandant of the Marine Corps HQMC, Code MPI-40 Washington, D. C. 22134	1
Koh Peng Kong OA Branch, DSO Ministry of Defense Blk 29 Middlesex Road Singapore 1024	1

	No. of Copies
Arthur P. Hunter, Jr. Professor and Chairman Department of Industrial Engineering and Management Sciences Northwestern University Evanston, IL 60201-9990	1
Major John R. Robinson Center for Naval Analysis MCOAG 4401 Ford Avenue Alexandria, VA 22302-0268	1
Commanding Officer ATTN: Mr. Murray Rowe Navy Personnel Research and Development Center San Diego, CA 92152	1
Commanding Officer ATTN: Barry Siegel Navy Personnel Research and Development Center San Diego, CA 92152	3
Captain Charles R. Dickinson Staff USCINCPAC Box 15 (J55) Camp H. M. Smith, HI 98661	1
Mr. R. Morton %DSAI 350 Fortune Terrance Rockville, MD 20854-2995	4
Library (Code 0142) Naval Postgraduate School Monterey, CA 93943-5000	2
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2

	No. of Copies
Office of Research Administration (Code 012) Naval Postgraduate School Monterey, CA 93943-5000	1
Center for Naval Analyses 4401 Ford Avenue Alexandria, VA 22302-0268	1
Library (Code 55) Naval Postgraduate School Monterey, CA 93943-5000	1
Operations Research Center, Rm E40-164 Massachusetts Institute of Technology Attn: R. C. Larson and J.F. Shapiro Cambridge, MA 02139	1
Institute for Defense Analysis 1800 North Beauregard Alexandria, VA 22311	1

DUDLEY KNOX LIBRARY



3 2768 00337473 7